Mathematics

Unit-I : Algebra, Number Theory and Linear Algebra

Group Theory : Groups, Subgroups, Normal Subgroups and Quotient Groups, Homomorphisms and applications, Permutation groups, Conjugacy and Class equation, Simple group, Sylow Theorms. Ring Theory : Rings, Special Classes of rings, Homomorphisms, Ideals and Quotient rings, Maximal and Prime ideals, Polynomial rings, Principal Ideal Domain, Unique Factorization Domain. Field : Field of Quotients of an Integral Domain, Polynomials over the rational field, Algebraic Extension of Fields: Irreducible polynomials and Eisenstein: Criterion, roots of Polynomial, Splitting field and its degree of extension, Multiple roots, Ruler and Compass Constructions, Symmetric function of roots, Solution of Cubic and Biquadratic Equations. Number Theory : Integers, g.c.d., Fundamental Theorem of Arithmetic, Euclidean Alogorithm, Arithmetical functions (Euler-function, Mobius function-), Dirichlet multiplication, Linear Congruences, Euler-Fermat Theorem, Linear Diophantine Equations, Fermat's Theorem, Fermat Little Theorem, Polynomial Congruence, Lagrange's Theorem, Chinese Remainder Theorem, Wilson's Theorem and Applications.

Vectorspace, Subspace, Linear Dependence , Independence, Dimension and Basis , Linear Transformation, Range and Kernel, Rank and Nullity, Inverse of Linear Transformation, Linear Map associated with matrix. Elementary Row Operations, Rank and Nullity of Matrix, Inverse of a Matr4ix, Determinants and product of Determinats, Eigen values, Eigen vectors, Characteristic roots. Canonical forms, Tringular form, Nilpotent Transformations, Similarity of Matrices, Quadratic form. Traces and Transpose, Hermitian, Unitary and Normal Transformation.

Unit-II : Real Analysis

Basic Topology : Finite, Countable and Uncountable sets, Metric Spaces, Topological Spaces, Basis, Closed sets, Open Sets, Limit Points, Properties of Connected Spaces and Compact Spaces, Heine Boril Theorem. Sequence and Series : Convergent Sequences, Subsequences, Convergence of Monotone Sequences, Couchy Sequences, Upper and Lower limits of Sequences, Bolzano Weirstrass Theorem, Series of non-negative terms, Convergence tests, Power Series, Couchy Convergence Criterion, Absolute Convergence, Alternating Series. Continuity and Differentiability : Properties of Continuous Function, Continuity and Compactness, Continuity and Connectedness, Discontinuity, Monotonic functions, Mean Value Theorem, Taylor Series. Function of Several Variables : Continuity Differentiability, Extreme Values, Maxima and Minima, Line Integral, Surface Integral, Volume Integral, Applications of Green's Theorem, Stokes Theorem and Gauss Theorem.

Riemann stieltjes integral Existence of the integral, Properties of the integral, Fundamental theorem of calculus, change of variables in on integral, Differentiation of integral. Sequence and series of functions Uniform convergence of sequence of functions, Cauchy criterion for uniform convergence, weierstrass test for uniform convergence, uniform convergence and continuity, uniform convergence and differentiation, construction of continuous function on the real line which is nowhere differentiable. Measure Theory Lebesgue outer measure , Properties of outer measure, Measurable sets, Cantor set, Borel set, and sets, Non measurable sets, Measurable functions, Properties of measurable functions. Lebesgue integration and lp spaces comparison of Lebesgue and Riemann integral, Lebesgue integral of bounded measurable functions over sets of finite measure, Bounded convergence theorem, Lebesgue integral for nonnegative measurable function. Fatou's Lemma, Monotone convergence theorem, lp spaces, essential supremum of a function, Minkowski and Holder inequalities, Absolute summable and summable series in a normal linear space completeness in lp.

Unit-III : Numerical Analysis and Differential Equations

Root Finding for Non-Linear Equations : Newton's Method, Secant Method, One-point Iteration Method, Multiple Roots, Newton Methods of Non-Linear Systems.
Interpolation Theory : Finite Differences, Newton's Forward and Backward differences, Newton's Divided differences, Lagrange's Interpolation, Errors in data and Forward differences, Hermite Interpolation, Piece-wise linear Interpolation.
Numerical Integration : Newton-cote integration formula, trapezoidal rule, Simpsons' rule, Gaussian quadrature, Asymptotic error formulas and their applications.
Numerical Methods for Ordinary Different Equations : Euler's Method, Multistep Methods, Midpoint Method, Trapezoidal Method, Single Step Method and Runge-Kutta Method.

Linear Differential Equations with constant coefficients and variable coefficients, system of Linear Differential Equations. Laplace Transformation : Linearity of the Laplace transformation. Laplace transforms of derivatives and integrals, shifting theorems. Differentiation and integration of transforms. Convolution theorem. Solution of integral equations and systems of differential equations using Laplace Transformation. Series Solution of differential equations: Power series method, Bessel, Legendre and Hypergeometirc equations. Bessel, Legendre functions and their properties . Sturm- Liouville problem, Orthogonality of eigen functions. Orthogonality of Bessel fuctions and Legendre polynomials.
Partial Differential Equations of the 1st order. Lagrange's solution some special types of equations, their solution, Charpit's general method of solution. Partial Differential Equations of second and Higher orders. Classification of linear partial differential equations of second order. Homogeneous and non-homogeneous equations with constant colffficients, Monge's method. Fourier Series and Fourier Transform, Convergence of Fourier series, Application of Fourier series and Fourier Transforms to Boundary value problems. Solution of Laplace equation, wave equation and heat conduction equations.

Unit-IV: Operation Research and Discrete Mathematics

Linear Programming : Simpler Method, Computational Procedure, Use of Artificial Variables. Duality in Linear Programming : General Primal-dual pair, Duality Theorems,Complementary Slackness Theorem, Duality and Simplex Method, Dual Simplex Method.
Games and Strategies : Two-person-Zero Sum Games, Minimax-Maximin Principle, Games with Saddle Points, Mixed Strategies, Graphical Solutions, Dominance Property, Arithmetic Method of nxn Games, General Solution of nxn rectangular Games. Transportation and

Assignment : General Transportation Problem, Finding Initial Basic Feasible Solution, Test of Optionality, Transportation Algorithm, Transhipment Problems. Mathematical Formulation of Assignment Problem, Method of Solution of Assignment Problem, Travelling Salesman Problem.

Fundamentals of logic, Normal forms, Logical Inferences, Methods of proof, Mathematical Induction, Rules of Inferences for quantified propositions. Lattice and Boolean Algebra – Binary relations, Equivalence relations, prset, Lattice, Hasse Diagram, Algebraic properties of Lattice, Paths and closures, Directed graphs and adjacency matrix, Boolean Algebra, Boolean functions, Minimization of Boolean functions. Recurrence relation – Generating functions of sequences, Calculating co-efficients of generating functions, Recurrence relation, solving recurrence relations by substitution and generating functions. Solution by the method of characteristic roots. Graph Theory – Trees and their properties, spanning trees, Binary trees, Euler's formula, Euler's circuits, Hamiltanian Graphs

Unit-V: Complex Analysis and Functional Analysis

Analytical Functions : Continuity, Differentiability, Couchy-Reimenn Equations, Analytic Functions, Harmonic Functions. Bilinear Transformation : Elementary Transformations, Bilinear Transformation, Mapping by Elementary Functions. Complex Integration : Couch's Theorem, Couchy's Integral Formula, Maximum Modulus Theorem, Liouville's Theorem, Morera's Theorem, Related Problems. Singularities and Calculus of residues : Series Expansion, Taylor's Series, Laurent's Series, Zeros of Analytic Function, Singularities, Residues, Councy's Residue Theorem, Evaluation of Definite Integrals.

Normed Linear space Linear spaces, Subspaes, Quotient spaces, properties of norm, Riesz Lemma, Continuity of linear maps, Bounded linear operations, Equivalent norms, Hahn Banach theorem and its consequences. Banach spaces Uniform boundedness principle, closed graph theorem and its consequences, open mapping theorem and its consequences. Spaces of Bounded linear functional Duals and transposes, Duals of lp, Lp[a,b], C[a,b], Weak convergence, weak* convergence, Reflexivity. Hilbert space Inner product spaces, Orthonormal sets, Gram Schmidt Orthonormalisation, Bessel's Inequality, Riesz. Fischer theorem, Projection theorem, Riesz representation theorem.

MATHEMATICS

MODEL QUESTIONS

➢ **Answer all the questions. Each question has four choices (a), (b), (c), (d) of which only one is the correct answer. Choose the choice (a), (b), (c) or (d) corresponding to the correct answer.**

Unit – I

1. Let U_n denote the integers relatively prime to n under multiplication mod n and U_n is a group. Then which of the following is not a cyclic group?
   (a) U_{17} (b) U_{18} (c) U_{20} (d) U_{25}
2. Let G be a group of order 30. Then which is true?
   (a) G has a normal subgroup of order 15.
   (b) A 3-Syllow subgroup of G is not normal in G.
   (c) There are 4 non-isomorphic groups of order 30.
   (d) None of these.
3. Which of the following is not a ring with respect to ordinary operation of addition and multiplication?
   (a) Z the set of all integers.
   (b) N the set of all natural numbers.
   (c) Q the set of rational numbers.
   (d) R the set of real numbers.
4. The eigen values of the matrix $\begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$ are –
   (a) {2 , 3} (b) {0 , 1} (c) {1 , 2} (d) {0 , 5}
5. Which of the following Diophantine equation has no solution?
   (a) $56x + 72y = 40$ (c) $24x + 138y = 18$
   (b) $18x + 3y = 48$ (d) $123x + 360y = 99$
6. An orthonormal set in an inner product space X is a –
   (a) Linearly independent set.
   (b) Linearly dependent set.
   (c) Linear span of X.
   (d) None of the above.

Unit – II
Analysis

7. A metric space is compact if it is –
   (a) Complete and totally bounded.
   (b) Not complete and totally bounded.
   (c) Complete and not totally bounded.
   (d) Neither complete nor bounded.
8. Which of the following statement is true?
   (a) The Lebesgue integrable function over (0 , 1) forms a vector space.
   (b) Continuous function over (0 , 1) is Lebesgue integrable.

(c) A function which is differentiable over (0 , 1) is Lebesgue integrable over (0 , 1).
(d) A function which is Lebesgue integrable is always Reimann Integrable.

9. Let { an } be the sequence of consecutive roots of the equation tan x = x for x>0. The value of $\lim_{n\to\infty} (a_{n+1} - a_n)$ is –
   (a) 0 (c) 2π
   (b) - π (d) π
10. The value of 1 - $1/_{7}$ + $1/_{9}$ - $1/_{15}$ + $1/_{17}$ - $1/_{23}$ + $1/_{25}$ - .... is –
   (a) /8 (√2 – 1)π (c) /4 (√2 – 1)π
   (b) /8 (√2 + 1)π (d) /4 (√2 + 1)π
11. Which of the following statement is true?
   (a) sin x is not uniformly continuous on [0 , ∞)
   (b) sin x^2 is uniformly continuous on [0 , ∞)
   (c) sin x^2 is not uniformly continuous on [0 , ∞)
   (d) none of (a), (b), (c) is true
12. every complete Metric Space is of –
   (a) Baire's first category.
   (b) Baire's second category.
   (c) Rare sets category.
   (d) None of these above.
13. If 1≤ x ≤ 64, the greatest value of $(\log_2 x)^4 + 12(\log_2 x)^2 \log_2 (8/x)$ is –
   (a) 18 (c) 81
   (b) 58 (d) 85

Unit – III
Numerical Analysis & Differential equation

14. The inverse Laplace transform of $\frac{3p - 2}{P^2 + 1}$ is –
   (a) 2cost + 3sint (c) cos2t + 3sin2t
   (b) $2t^2$ + 3sin2t (d) 3cost – 2sint
15. The equation $xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$ is –
   (a) Parabolic (c) Elliptic
   (b) Hyperbolic (d) None of these.
16. Her mite equation is given by –
   (a) $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$
   (b) $y'' - 2xy' + 2py = 0$
   (c) $y'' + p(x)\, dy/dx + Q(x)y = 0$
   (d) $x^2y'' + xy' + (x^2 - p^2)y = 0$
17. The Her mite interpolating polynomial with n nodes is of degree at most –
   (a) 2n (c) 2n + 1
   (b) 2n – 1 (d) $\frac{n}{2}$
18. If the rate of Convergence of Muller method for finding the roots of the equation f(x) = 0 is, then the value of p is –
   (a) 1.62 (c) 2
   (b) 1.84 (d) 1

Unit – IV

19. The set of all feasible solutions to a LPP $A_x = b$; $x>0$ is –
   (a) Neither open nor closed.
   (b) Open convex set
   (c) Closed convex set
   (d) None of these
20. No. of basic variables in a balanced transportation problem with m origins and n destinations is –
   (a) m + n (c) m + n + 1
   (b) m + n – 1 (d) m – n
21. Solving a LPP by two phase method in Phase-I, if the optimal value of auxiliary objective function be not zero then the problem have –
   (a) No feasible solution
   (b) Only one feasible solution
   (c) Infinite number of F.S.
   (d) None of these
22. A tree has two vertices of degree 2, one vertex of degree 3 and 3 vertices of degree four. Then the number of vertices of degree 1 is –
   (a) 9 (c) 8
   (b) 6 (d) 11
23. The representation of the Boolean function f(x , y) = x + xy in disjunctive normal form is –
   (a) xy + xy' (c) xy + x'y
   (b) x'y' + x'y (d) x'y' + xy'
24. The graph contains 16 edges and all vertices of degree 2. Then the number of vertices of the graph is –
   (a) 16 (c) 12
   (b) 8 (d) 24

Unit – V

25. The residue of $\frac{z3}{z2-1}$ at z = ∞
   (a) -1 (b) 1 (c) $\frac{1}{2}$ (d) 0
26. For the function $e^z/z \sin mz$, z = 0 is a –
   (a) Essential singularity (c) Removable singularity
   (b) Pole (d) Simple zero
27. The function f(z) is meromorphic function if –
   (a) $f(z) = \frac{ez}{z}$ (c) $f(z) = \sin\frac{1}{z}$
   (b) $f(z) = e^{1/z}$ (d) $f(z) = (z – i) \sin \frac{1}{z+2i}$
28. If x is a Hausdorff Space then a sequence of points of x converges to –
   (a) Infinitely many points of x.
   (b) At most one point of x.
   (c) At most two points of x.
   (d) None of the above.
29. Every finite dimensional normed space is –
   (a) Transitive (c) Non-reflexive
   (b) Reflexive (d) None of these
30. Every closed subspace of a normed space is –

(a) Weekly complete
(b) Weekly sequentially compact
(c) Weekly closed
(d) None of the above

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THE END